Homework 2, ORFE 569

Due Mar. 6, 2007

1. Suppose that X_t has finite q-th variation ($q > 0$). Show that for $p \in (0, q)$, the p-th variation of X_t is infinity; and for $p > q$, the p-th variation of X_t is zero.

2. For Double Exponential Jump-Diffusion (DEJD) model (Kou 2002, *Management Sciences*, or Ramezani and Zeng 2007, *Annals of Finance*):

$$\frac{dX(t)}{X(t-)} = \mu dt + \sigma dB(t) + \sum_{j=u,d} (V^j_{N^j(\lambda^j t)} - 1) dN^j(\lambda^j t)$$

where μ and σ are the drift and volatility terms, B(t) is a standard Wiener process, V^j is the jump magnitude, and $N^j(\lambda^j)$ are independent Poisson processes with intensity parameters λ^j ($j = u, d$ represent up- and down-jumps respectively). It is assumed that the up-jump magnitudes (V^u) are distributed Pareto(η_u) with density function $f_{V^u}(x)$=($\frac{\eta_u}{x^{\eta_u+1}}$) where $V^u \geq 1$. Similarly, the down-jump magnitudes (V^d) are distributed Beta(η_d,1) with density function $f_{V^d}(x) = \eta_d x^{\eta_d - 1}$ where $0 < V^d < 1$. All jumps are assumed to be independent. This form is used in Ramezani and Zeng (2007).

   (a) Determine the means and variances of V^u and V^d.

   (b) Show that

$$S(t) = S(0) \exp\{(\mu - \frac{1}{2}\sigma^2)t + \sigma Z(t)\} \prod_{j=u,d} V^j(N(\lambda^j t))$$

   where

$$V^j(N(\lambda^j t)) = \begin{cases} 1 & \text{if } N(\lambda^j t) = 0 \\ \prod_{i=1}^{N(\lambda^j t)} V^j_i & \text{if } N(\lambda^j t) = 1, 2, 3, \ldots \end{cases}$$

   (c) Determine (show details how to obtain) its generator.

   (d) (**Extra Credits:**) Show this form is equivalent in distribution with that in Kuo 2002.

3. Determine (show details how to obtain) the generator for the following Jumping Stochastic Volatility Linear Brownian Motion (JSV-LBM) model:

$$\begin{cases} dX(t) = \mu dt + \sigma(t) dW(t), \\ d\sigma(t) = \big(J_{N(t)} - \sigma(t-)\big) dN(t), \end{cases} \tag{1}$$

where $W(t)$ is a standard Brownian motion, $N(t)$ is a Poisson process with in tensity λ and is independent of $W(t)$, and $\{J_i\}$ is a sequence of i.i.d. random variables with density $g(z)$ and they are independent of $W(t)$ and $N(t)$.

4. Let $U = \{1, 2, ..., N\}$ represent all possible regimes and $S(t)$ be the most recent regime. $m(t, \cdot)$ is a marked point process with the stochastic intensity kernel,

$$\gamma_m(dt, du) = h(u, S(t-); X(t)) \eta(du) dt$$

where $h(u, S(t-); X(t))$ is the conditional regime-shift (from $S(t-)$ to u) intensity at time t, and η is counting measure on U. $h(u, S(t-); X(t))$ is assumed to be bounded.

Determine (show details how to obtain) the generator for the following *regime-switching* diffusion model (Wu and Zeng 2007):

$$\begin{cases} dX_t = \mu(S_t, X_t)dt + \sigma(S_t, X_t)dW_t \\ dS_t = \int_U (u - S(t-))m(dt, du). \end{cases} \tag{2}$$