Homework 4, ORFE 569

Due Apr. 24, 2007

1. Let $\phi(f,t)$ as defined in Section 3.1 and satisfy the unnormalized filtering equation (3.1) in Zeng (2003). Let $l(f,t) = \ln(\phi(f,t))$ and derive the SDE for $l(f,t)$ using Itô formula for semimartingale. (Note that $l(1,t)$ becomes the log-likelihood.)

2. Let $L(t) = \frac{dP}{dQ}\Big|_{\mathcal{F}_t}$. Show that Z is a P-local martingale if and only if LZ is a Q - local martingale. (Hint: Use Bayes Theorem)

3. (Lemma A.3 in Zeng 2003) Suppose that $\vec{X}$ and $\vec{Y}$ are independent. If U is $\mathcal{F}_t^{\vec{X},\vec{Y}}$-adapted, satisfying $\int_0^t E[|U(s)|]ds < \infty$, then

$$E^Q\Big[\int_0^t U(s)ds|\mathcal{F}_t^{\vec{Y}}\Big] = \int_0^t E^Q[U(s)|\mathcal{F}_s^{\vec{Y}}]ds.$$

4. Derive the recursive algorithm to compute the Bayes factor for the model selection of your model for Lab Assignments 2 and 3 verse another model of your choice by do the following: (Section 4 in Kourtizin and Zeng 2004 gives an example)

   (a) Write down the two models. One in the form of *Filtering with counting process observations* and in the form of *Construction of Price from Intrinsic Value.* Specify $p(y_j|x)$ also for each model.

   (b) Write down the generators for the two models. $\mathbf{A}^k f_k$ for $k = 1, 2$.

   (c) Write down the generators for the approximate models, $\mathbf{A}_\varepsilon^{(k)} f_k$ for $k = 1, 2$.

   (d) Define the appropriate $q_{\varepsilon,t}^{(k)}$, and $q_\varepsilon^{(k)}(\cdots;t)$ for $k = 1, 2$.

   (e) Define the appropriate lattice-point indicators for Model k.

   (f) Derive (in detail) the two propagation parts of the recursive algorithm for $q_\varepsilon^{(k)}(\cdots;t_{i+1}-)$ for $k = 1, 2$.

   (g) Derive the two updating parts of the recursive algorithm for $q_\varepsilon^{(k)}(\cdots;t_{i+1})$ for $k = 1, 2$.

   (h) Write down the equations for $B_{12}(t_{i+1})$ and $B_{21}(t_{i+1})$.

   (i) Write appropriate priors for the two models.